Chapter 28A - Direct Current Circuits

A PowerPoint Presentation by

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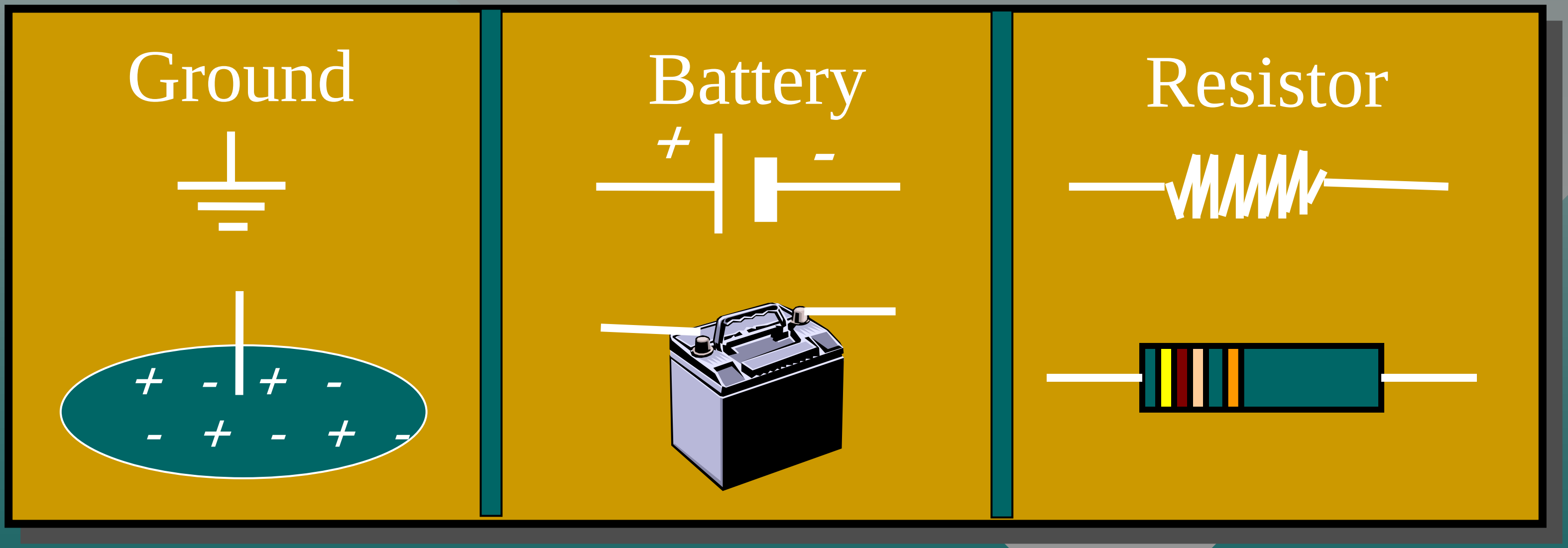
Objectives: After completing this module, you should be able to:

- Determine the effective resistance for a number of resistors connected in series and in parallel.
- For simple and complex circuits, determine the voltage and current for each resistor.
- Apply Kirchoff's laws to find currents and voltages in complex circuits.

Electrical Circuit Symbols

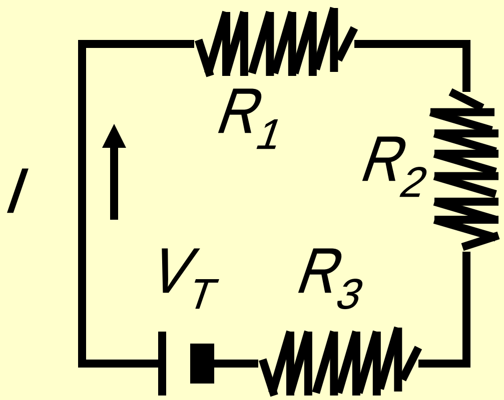
Electrical circuits often contain one or more resistors grouped together and attached to an energy source, such as a battery.

The following symbols are often used:



Resistances in Series

Resistors are said to be connected in series when there is a single path for the current.



Only one current

The current I is the same for each resistor R_1, R_2 and R_3.

The energy gained through $\mathcal{E}$ is lost through R_1, R_2 and R_3.

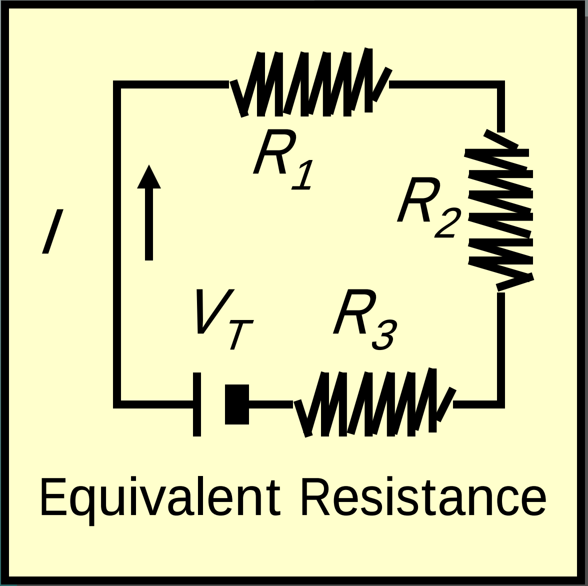
The same is true for voltages:

For series connections:

$$I = I_1 = I_2 = I_3$$
$$V_T = V_1 + V_2 + V_3$$

Equivalent Resistance: Series

The equivalent resistance R_e of a number of resistors connected in series is equal to the sum of the individual resistances.



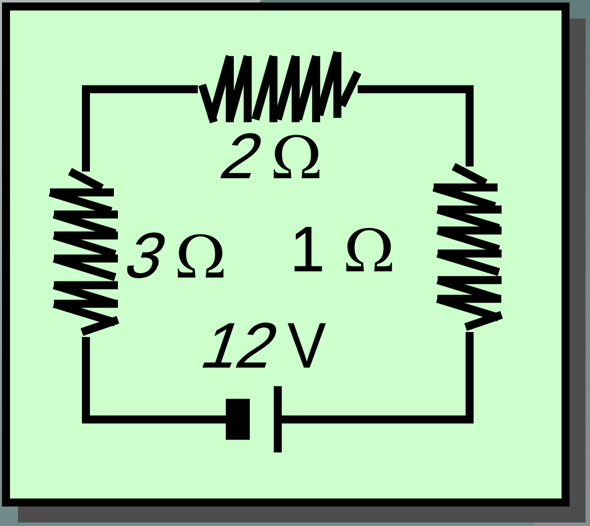
$$V_T = V_1 + V_2 + V_3 \; ; \quad (V = IR)$$

$$I_T R_e = I_1 R_1 + I_2 R_2 + I_3 R_3$$

$$\text{But} \; . \; . \; . \; I_T = I_1 = I_2 = I_3$$

$$R_e = R_1 + R_2 + R_3$$

Example 1: Find the equivalent resistance R_e. What is the current I in the circuit?



$$R_e = R_1 + R_2 + R_3$$

$$R_e = 3\ \Omega + 2\ \Omega + 1\ \Omega = 6\ \Omega$$

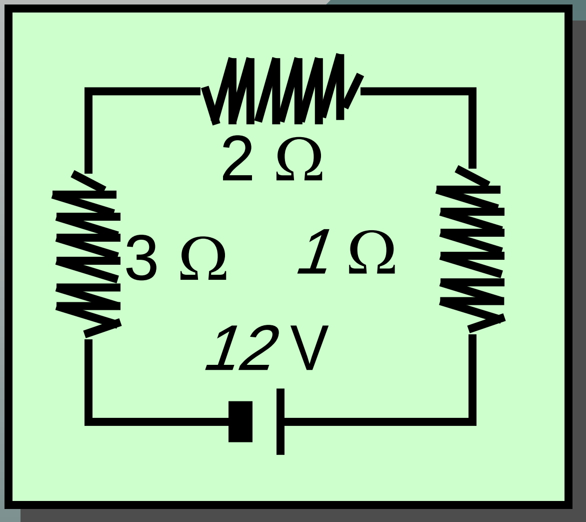
Equivalent $R_e = 6\ \Omega$

The current is found from Ohm's law: $V = IR_e$

$$I = \frac{V}{R_e} = \frac{12\text{ V}}{6\ \Omega}$$

I = 2 A

Example 1 (Cont.): Show that the voltage drops across the three resistors totals the 12-V emf.



$R_e = 6\ \Omega$

I = 2 A

Current I = 2 A same in each R.

$V_1 = IR_1;\ \ V_2 = IR_2;\ V_3 = IR_3$

V_1 = (2 A)(1 Ω) = 2 V

V_1 = (2 A)(2 Ω) = 4 V

V_1 = (2 A)(3 Ω) = 6 V

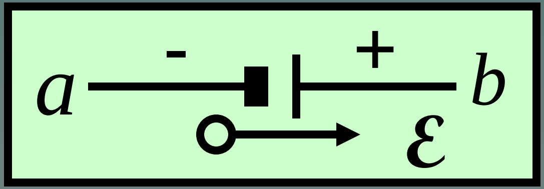
$V_1 + V_2 + V_3 = V_T$

2 V + 4 V + 6 V = 12 V

Check !

Sources of EMF in Series

The output direction from a source of emf is from + side:



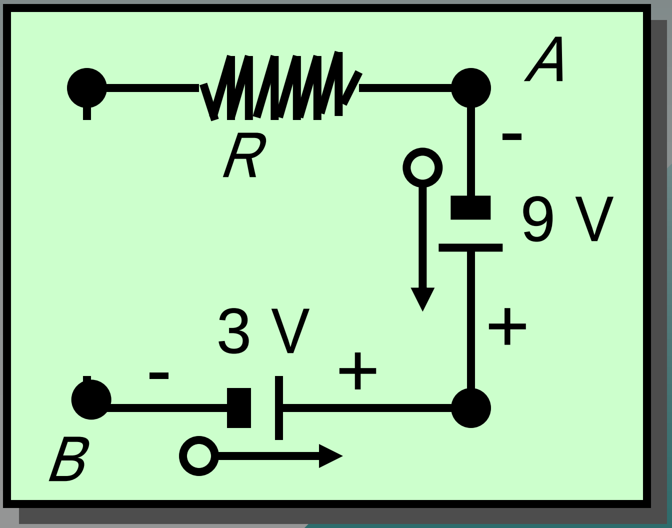
Thus, from ***a*** to *b* the potential increases by $\mathcal{E}$;

From *b* to ***a***, the potential decreases by $\mathcal{E}$.

Example: Find ΔV for path AB and then for path BA.

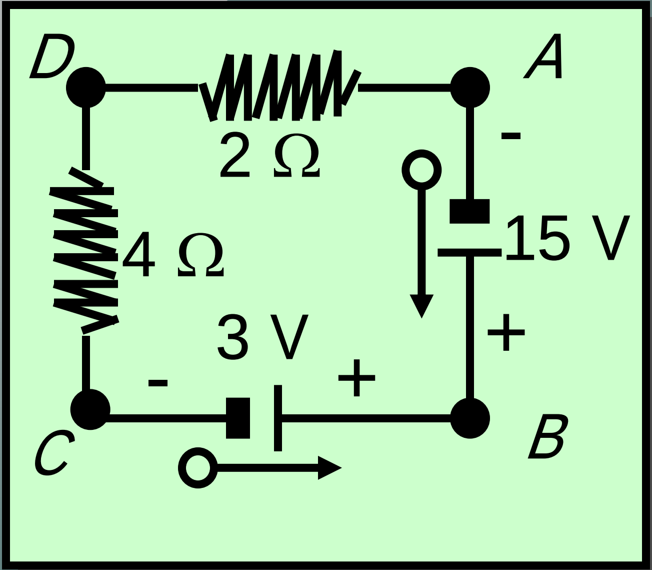
AB: ΔV = +9 V – 3 V = +6 V

BA: ΔV = +3 V - 9 V = -6 V



A Single Complete Circuit

Consider the simple series circuit drawn below:



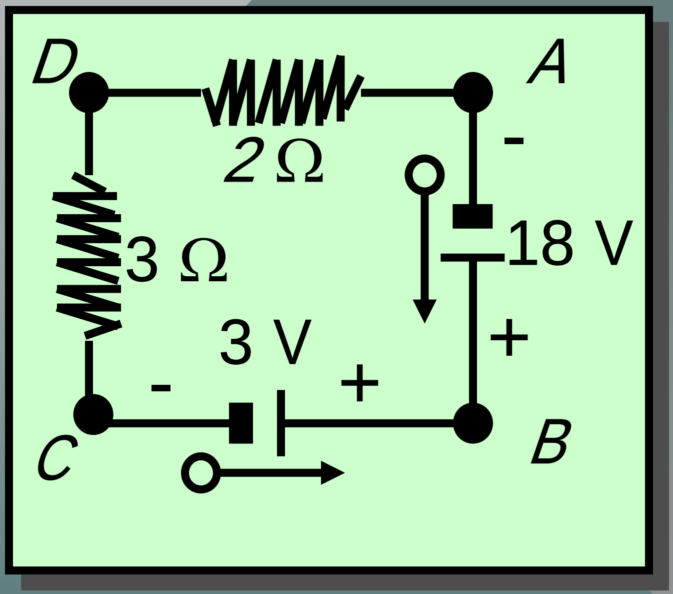
Path ABCD: Energy and V increase through the 15-V source and decrease through the 3-V source.

$$\Sigma\mathcal{E} = 15\text{ V} - 3\text{ V} = 12\text{ V}$$

The net gain in potential is lost through the two resistors: these voltage drops are IR_2 and IR_4, so that the sum is zero for the entire loop.

Finding I in a Simple Circuit.

<u>Example 2:</u> Find the current *I* in the circuit below:



$$\Sigma \mathcal{E} = 18\text{ V} - 3\text{ V} = 15\text{ V}$$

$$\Sigma R = 3\ \Omega + 2\ \Omega = 5\ \Omega$$

Applying Ohm's law:

$$I = \frac{\Sigma \mathcal{E}}{\Sigma R} = \frac{15\text{ V}}{5\ \Omega}$$

I = 3 A

In general for a single loop circuit:

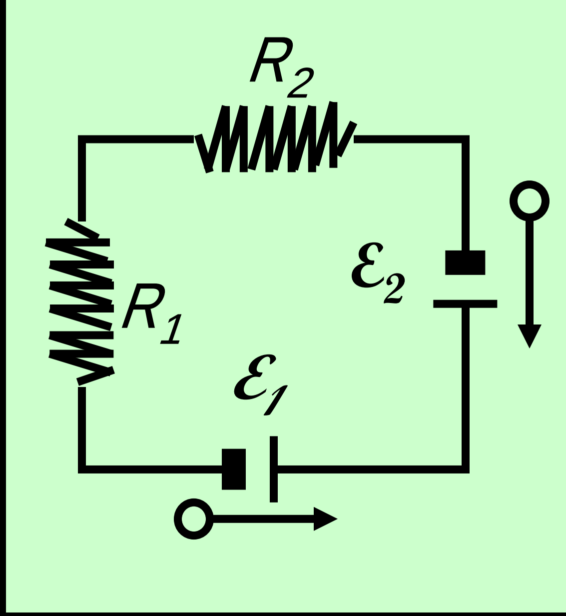
$$I = \frac{\Sigma \mathcal{E}}{\Sigma R}$$

Summary: Single Loop Circuits:

Resistance Rule: $R_e = \Sigma R$

Current: $$I = \frac{\Sigma \mathcal{E}}{\Sigma R}$$

Voltage Rule: $\Sigma \mathcal{E} = \Sigma IR$



Complex Circuits

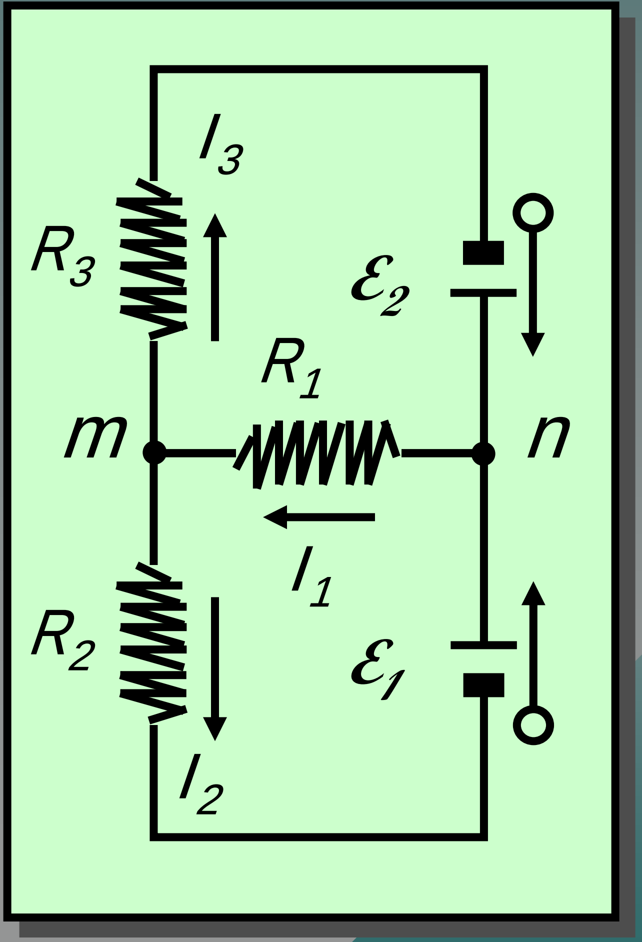
A complex circuit is one containing more than a single loop and different current paths.

At junctions m and n:

$I_1 = I_2 + I_3$ or $I_2 + I_3 = I_1$

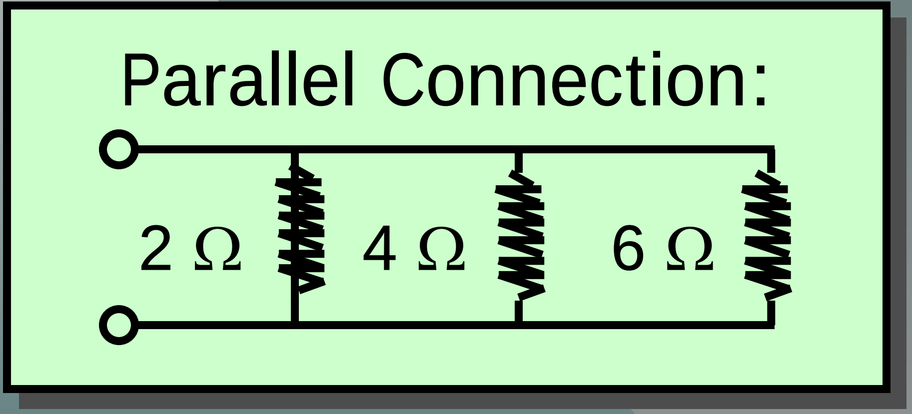
Junction Rule:

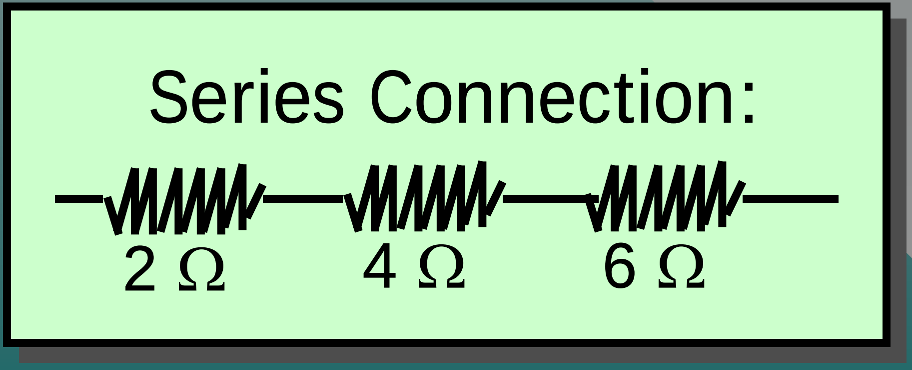
$\Sigma I \text{ (enter)} = \Sigma I \text{ (leaving)}$



Parallel Connections

Resistors are said to be connected in parallel when there is more than one path for current.





For Parallel Resistors:

$$V_2 = V_4 = V_6 = V_T$$

$$I_2 + I_4 + I_6 = I_T$$

For Series Resistors:

$$I_2 = I_4 = I_6 = I_T$$

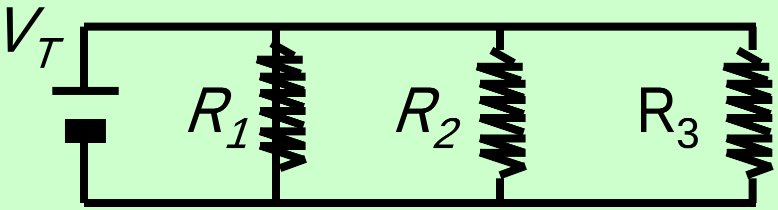
$$V_2 + V_4 + V_6 = V_T$$

Equivalent Resistance: Parallel

$V_T = V_1 = V_2 = V_3$

$I_T = I_1 + I_2 + I_3$

Parallel Connection:



Ohm's law: $I = \frac{V}{R}$

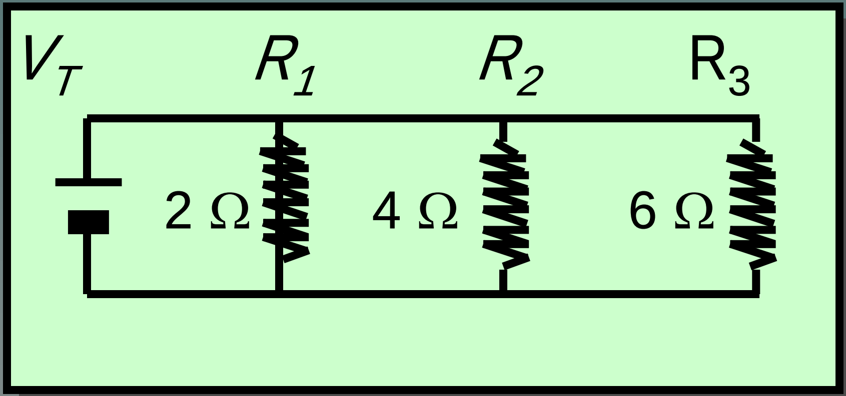
$$\frac{V_T}{R_e} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \Rightarrow \frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The equivalent resistance for Parallel resistors:

$$\frac{1}{R_e} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Example 3. Find the equivalent resistance R_e for the three resistors below.

$$\frac{1}{R_e} = \sum_{i=1}^{N} \frac{1}{R_i}$$



$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

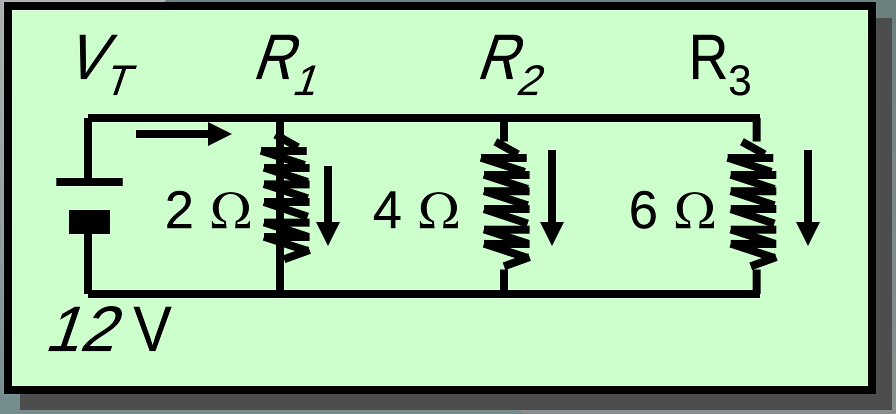
$$\frac{1}{R_e} = \frac{1}{2\Omega} + \frac{1}{4\Omega} + \frac{1}{6\Omega} = 0.500 + 0.250 + 0.167$$

$$\frac{1}{R_e} = 0.917; \quad R_e = \frac{1}{0.917} = 1.09\,\Omega$$

$R_e = 1.09\ \Omega$

For parallel resistors, R_e is less than the least R_i.

Example 3 (Cont.): Assume a 12-V emf is connected to the circuit as shown. What is the total current leaving the source of emf?



V_T = 12 V; R_e = 1.09 Ω

$V_1 = V_2 = V_3 = 12$ V

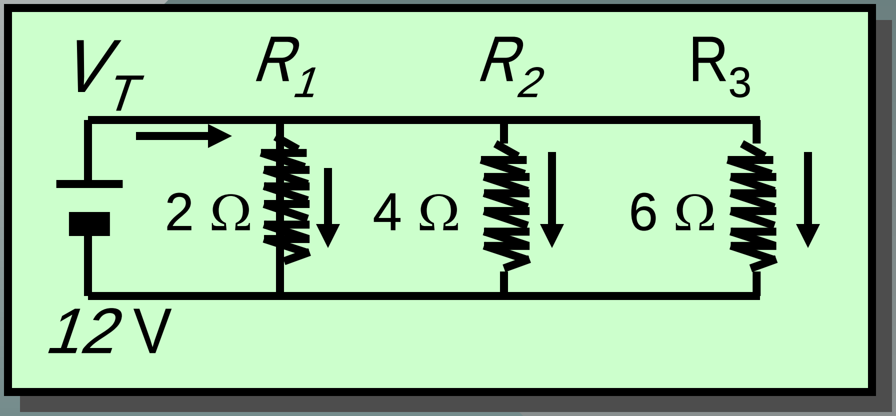
$I_T = I_1 + I_2 + I_3$

Ohm's Law: $I = \frac{V}{R}$

$$I_e = \frac{V_T}{R_e} = \frac{12\text{ V}}{1.09\ \Omega}$$

Total current: I_T = 11.0 A

Example 3 (Cont.): Show that the current leaving the source I_T is the sum of the currents through the resistors R_1, R_2, and R_3.



I_T = 11 A; R_e = 1.09 Ω

$V_1 = V_2 = V_3 = 12$ V

$I_T = I_1 + I_2 + I_3$

$$I_1 = \frac{12\text{ V}}{2\ \Omega} = 6\text{ A} \qquad I_2 = \frac{12\text{ V}}{4\ \Omega} = 3\text{ A} \qquad I_3 = \frac{12\text{ V}}{6\ \Omega} = 2\text{ A}$$

6 A + 3 A + 2 A = 11 A

Check !

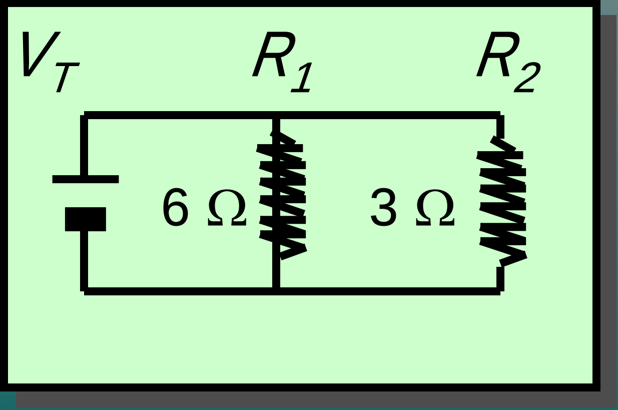
Short Cut: Two Parallel Resistors

The equivalent resistance R_e for two parallel resistors is the product divided by the sum.

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2};$$

$$R_e = \frac{R_1 R_2}{R_1 + R_2}$$

Example:

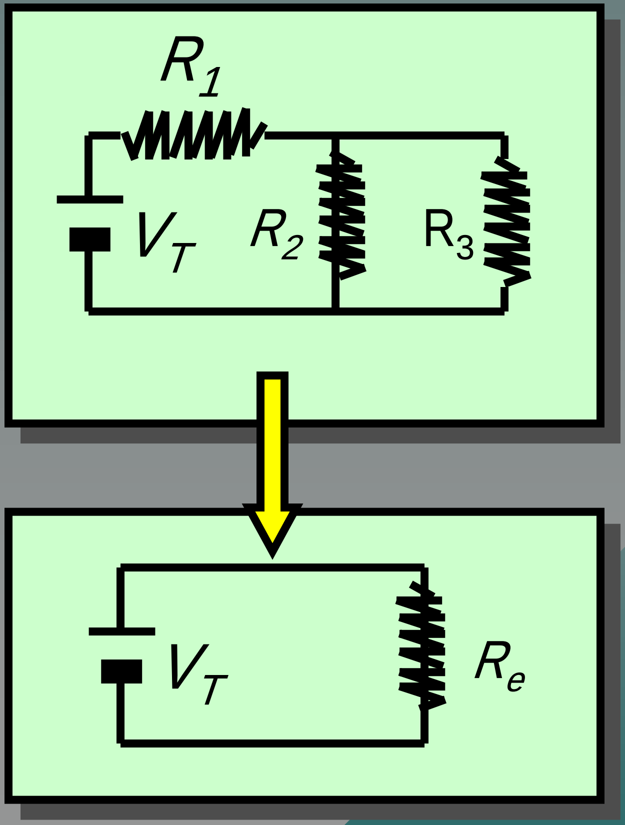


$$R_e = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega}$$

$R_e = 2\ \Omega$

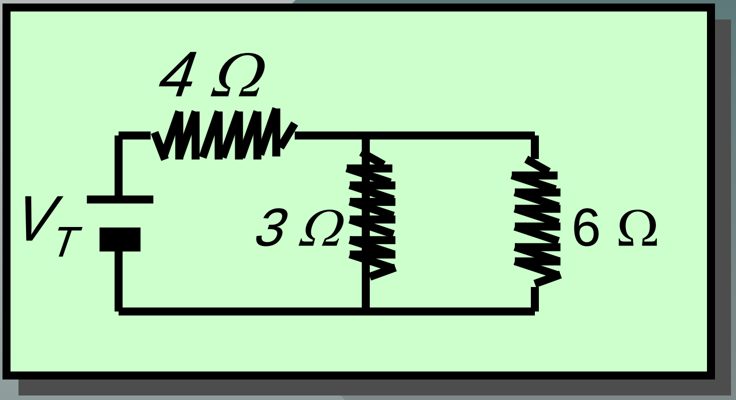
Series and Parallel Combinations

In complex circuits resistors are often connected in both series and parallel.



In such cases, it's best to use rules for series and parallel resistances to reduce the circuit to a simple circuit containing one source of emf and one equivalent resistance.

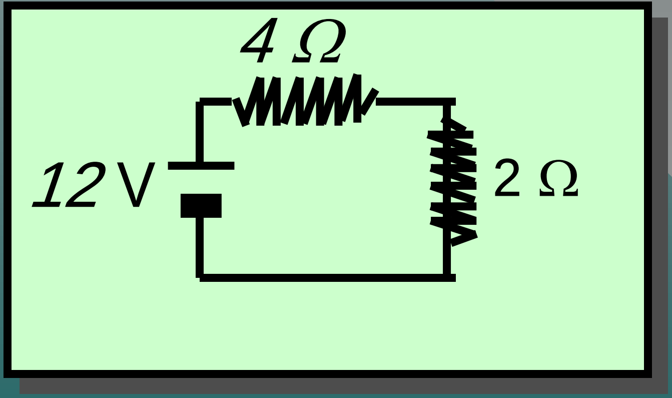
Example 4. Find the equivalent resistance for the circuit drawn below (assume V_T = 12 V).

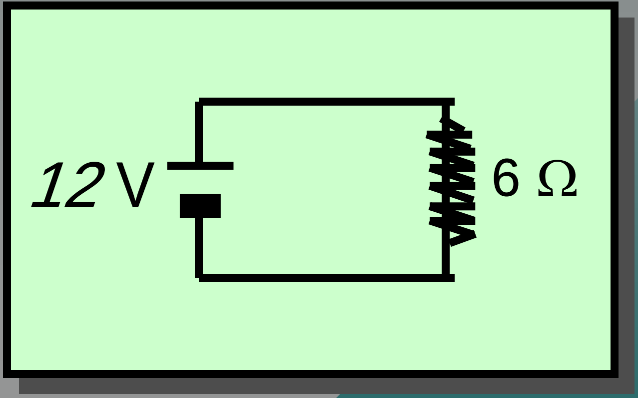


$$R_{3,6} = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega} = 2\Omega$$

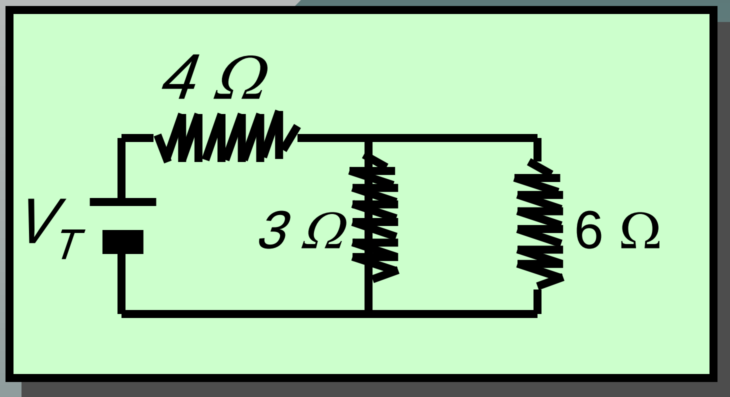
$$R_e = 4\ \Omega + 2\ \Omega$$

$$R_e = 6\ \Omega$$





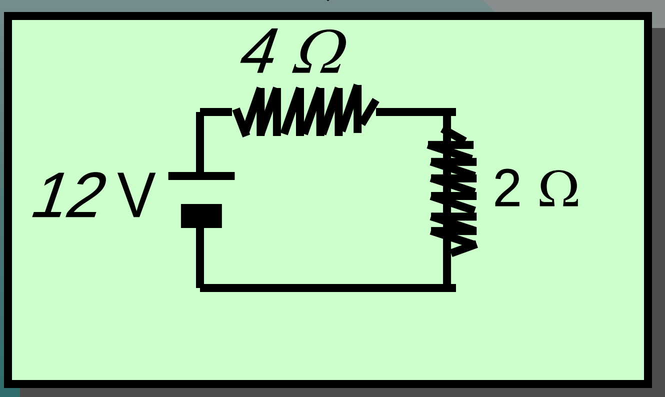
Example 3 (Cont.) Find the total current I_T.

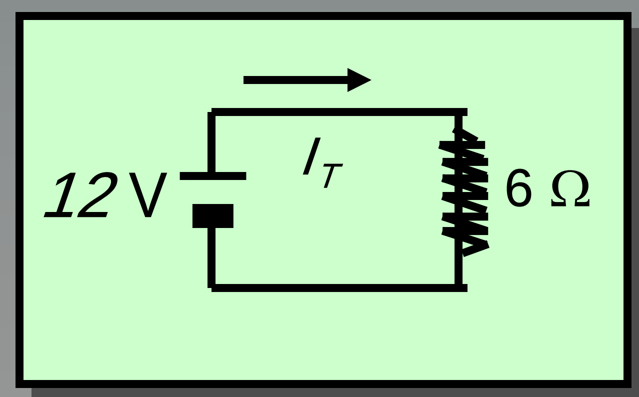


$R_e = 6\ \Omega$

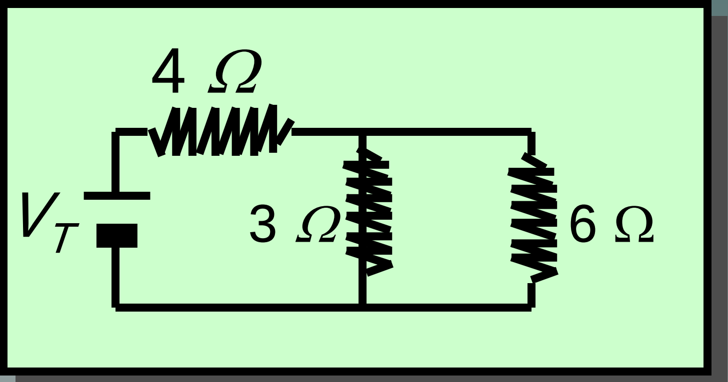
$$I = \frac{V_T}{R_e} = \frac{12\ \text{V}}{6\ \Omega}$$

$I_T = 2.00\ \text{A}$





Example 3 (Cont.) Find the currents and the voltages across each resistor.



$I_4 = I_T = 2$ A

$V_4 = (2\text{ A})(4\ \Omega) = 8\text{ V}$

The remainder of the voltage: (12 V – 8 V = 4 V) drops across EACH of the parallel resistors.

$V_3 = V_6 = 4$ V

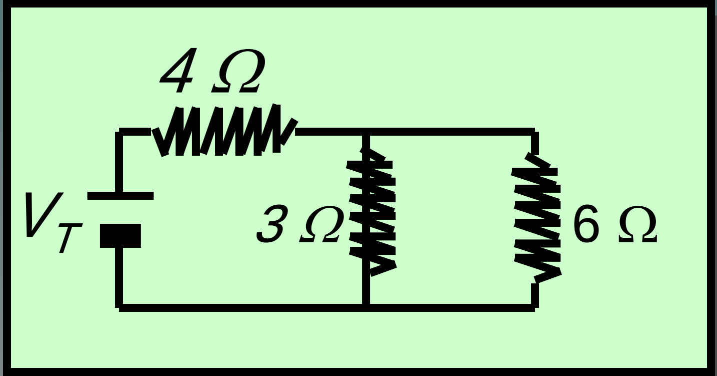
This can also be found from $V_{3,6} = I_{3,6}R_{3,6} = (2\text{ A})(2\ \Omega)$

(Continued . . .)

Example 3 (Cont.) Find the currents and voltages across each resistor.

$V_4 = 8\text{ V}$

$V_6 = V_3 = 4\text{ V}$



$$I_3 = \frac{V_3}{R_3} = \frac{4\text{V}}{3\Omega}$$

$I_3 = 1.33\text{ A}$

$$I_6 = \frac{V_6}{R_6} = \frac{4\text{V}}{6\Omega}$$

$I_6 = 0.667\text{ A}$

$I_4 = 2\text{ A}$

Note that the junction rule is satisfied:

$\Sigma I\ (\text{enter}) = \Sigma I\ (\text{leaving})$

$I_T = I_4 = I_3 + I_6$

Kirchoff's Laws for DC Circuits

Kirchoff's first law: The sum of the currents entering a junction is equal to the sum of the currents leaving that junction.

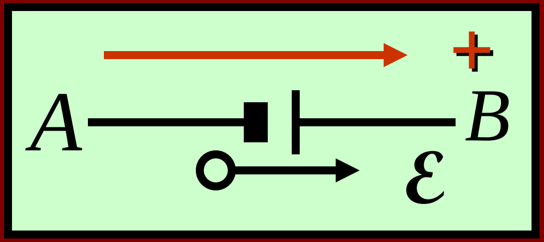
Junction Rule: ΣI *(enter)* $= \Sigma I$ *(leaving)*

Kirchoff's second law: The sum of the emf's around any closed loop must equal the sum of the IR drops around that same loop.

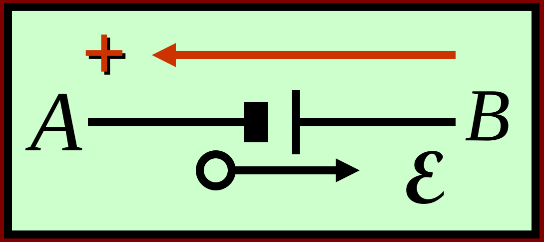
Voltage Rule: $\Sigma\mathcal{E} = \Sigma IR$

Sign Conventions for Emf's

- When applying Kirchoff's laws you must assume a consistent, positive tracing direction.
- When applying the voltage rule, emf's are positive if normal output direction of the emf is with the assumed tracing direction.
- If tracing from A to B, this emf is considered positive.



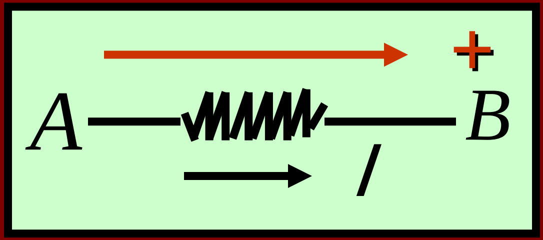
- If tracing from B to A, this emf is considered negative.



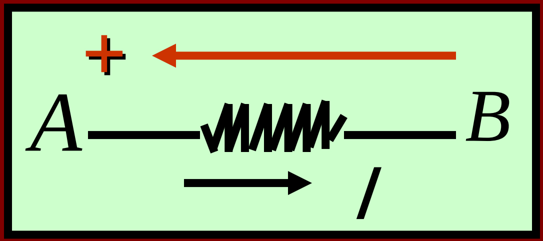
Signs of IR Drops in Circuits

- When applying the voltage rule, IR drops are positive if the assumed current direction is with the assumed tracing direction.

- If tracing from A to B, this IR drop is positive.

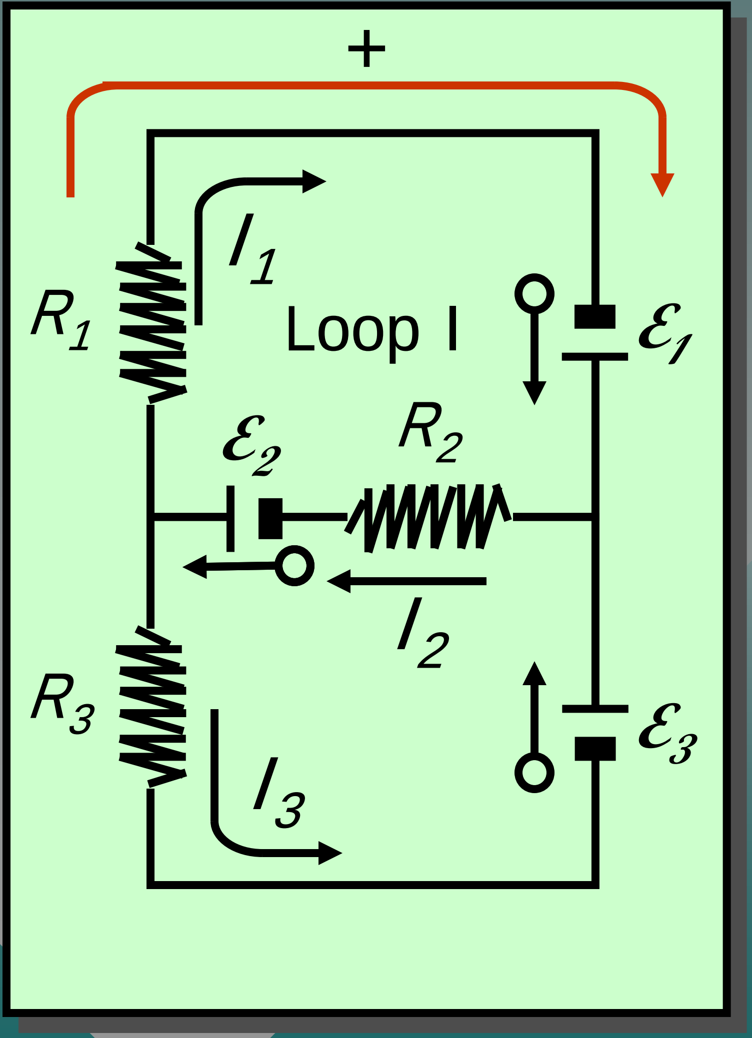


- If tracing from B to A, this IR drop is negative.



Kirchoff's Laws: Loop I

1. Assume possible consistent flow of currents.
2. Indicate positive output directions for emf's.
3. Indicate consistent tracing direction. (clockwise)



Junction Rule: $I_2 = I_1 + I_3$

Voltage Rule: $\Sigma\mathcal{E} = \Sigma IR$

$$\mathcal{E}_1 + \mathcal{E}_2 = I_1R_1 + I_2R_2$$

Kirchoff's Laws: Loop II

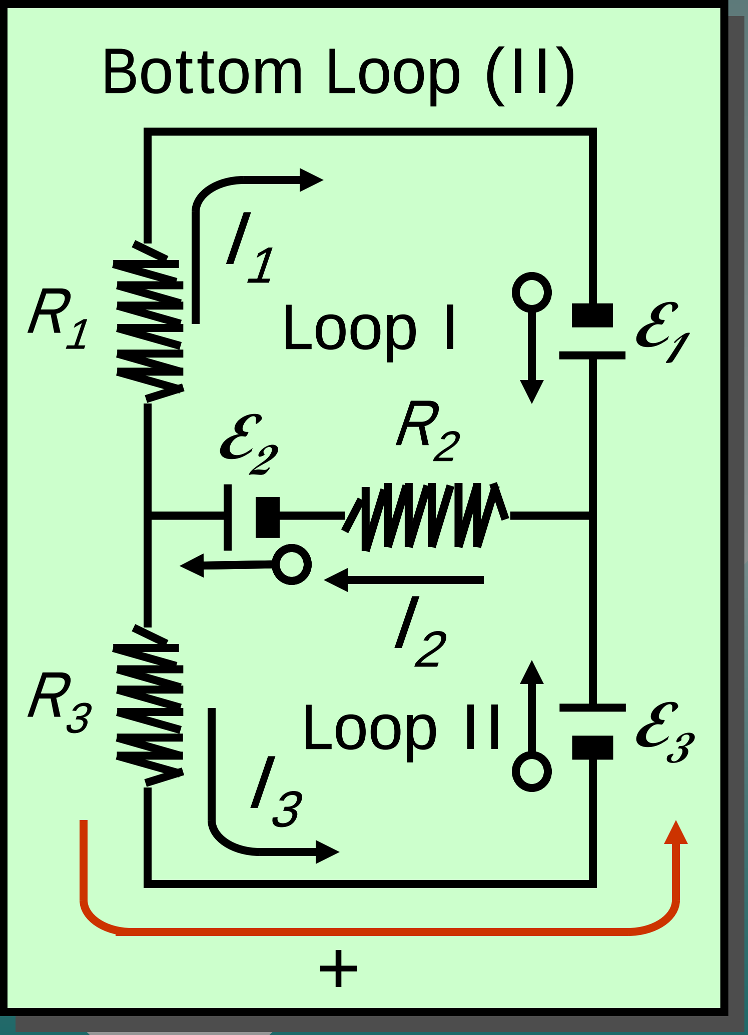
4. Voltage rule for Loop II: Assume counterclockwise positive tracing direction.

Voltage Rule: $\Sigma\mathcal{E} = \Sigma IR$

$$\mathcal{E}_2 + \mathcal{E}_3 = I_2R_2 + I_3R_3$$

Would the same equation apply if traced clockwise?

Yes! $- \mathcal{E}_2 - \mathcal{E}_3 = -I_2R_2 - I_3R_3$



Kirchoff's laws: Loop III

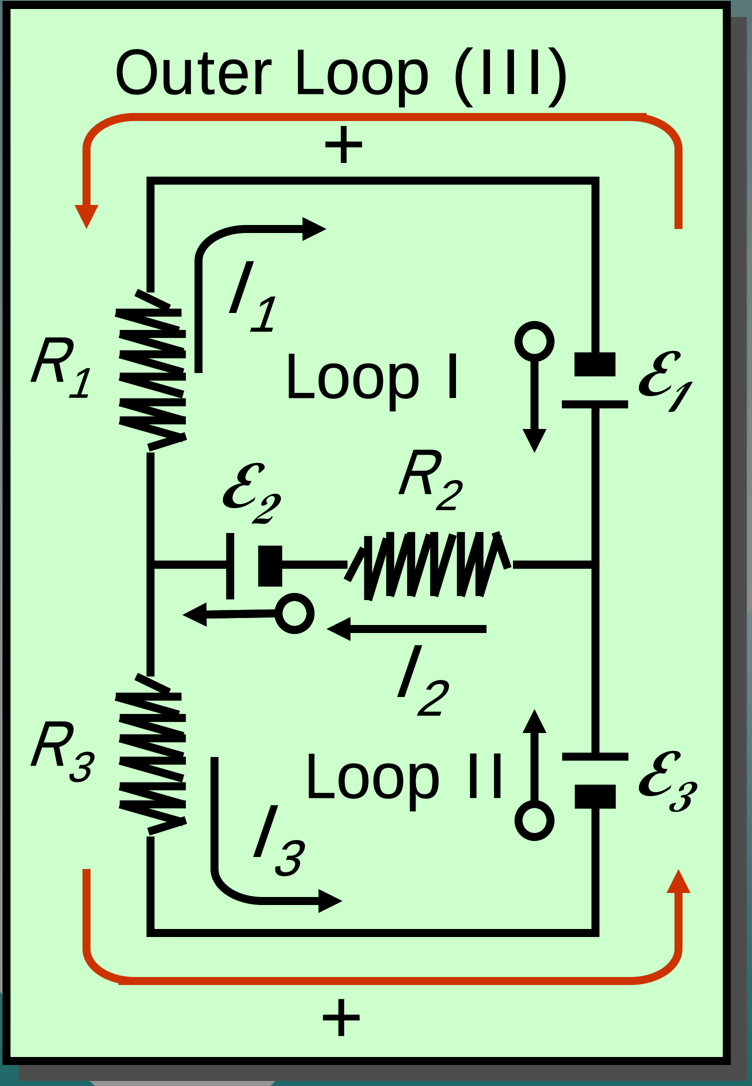
5. Voltage rule for Loop III: Assume counterclockwise positive tracing direction.

Voltage Rule: $\Sigma\mathcal{E} = \Sigma IR$

$$\mathcal{E}_3 - \mathcal{E}_1 = -I_1R_1 + I_3R_3$$

Would the same equation apply if traced clockwise?

Yes! $\mathcal{E}_3 - \mathcal{E}_1 = I_1R_1 - I_3R_3$



Four Independent Equations

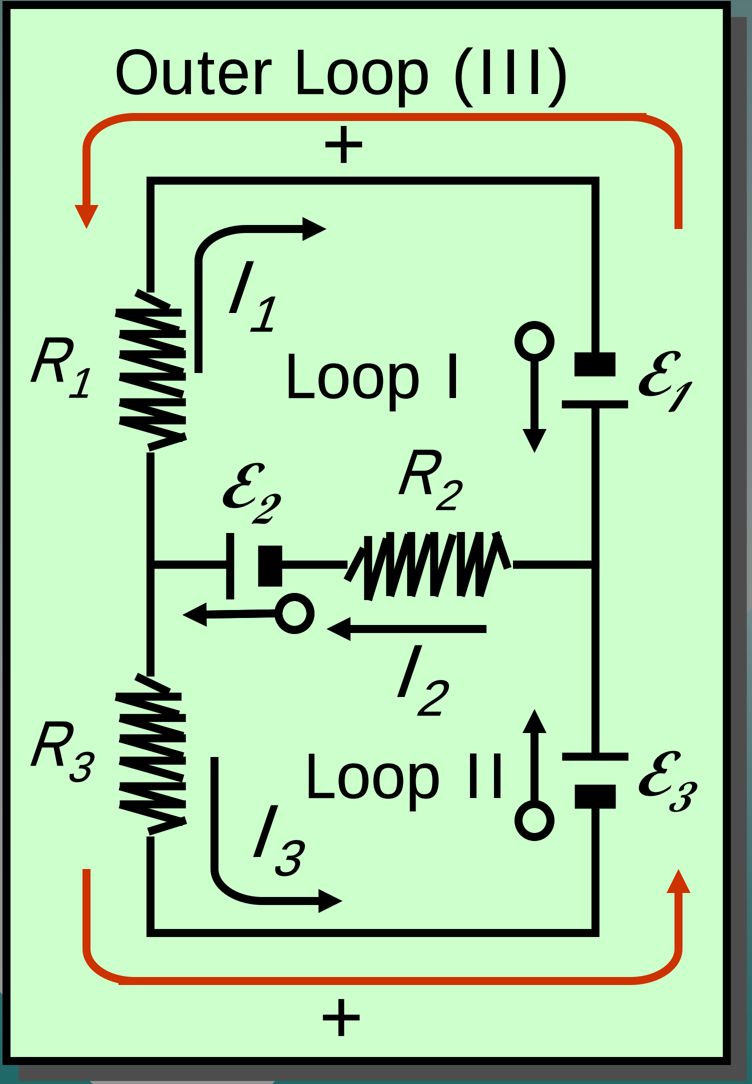
6. Thus, we now have four independent equations from Kirchoff's laws:

$$I_2 = I_1 + I_3$$

$$\mathcal{E}_1 + \mathcal{E}_2 = I_1R_1 + I_2R_2$$

$$\mathcal{E}_2 + \mathcal{E}_3 = I_2R_2 + I_3R_3$$

$$\mathcal{E}_3 - \mathcal{E}_1 = -I_1R_1 + I_3R_3$$



Example 4. Use Kirchoff's laws to find the currents in the circuit drawn to the right.

Junction Rule: $I_2 + I_3 = I_1$

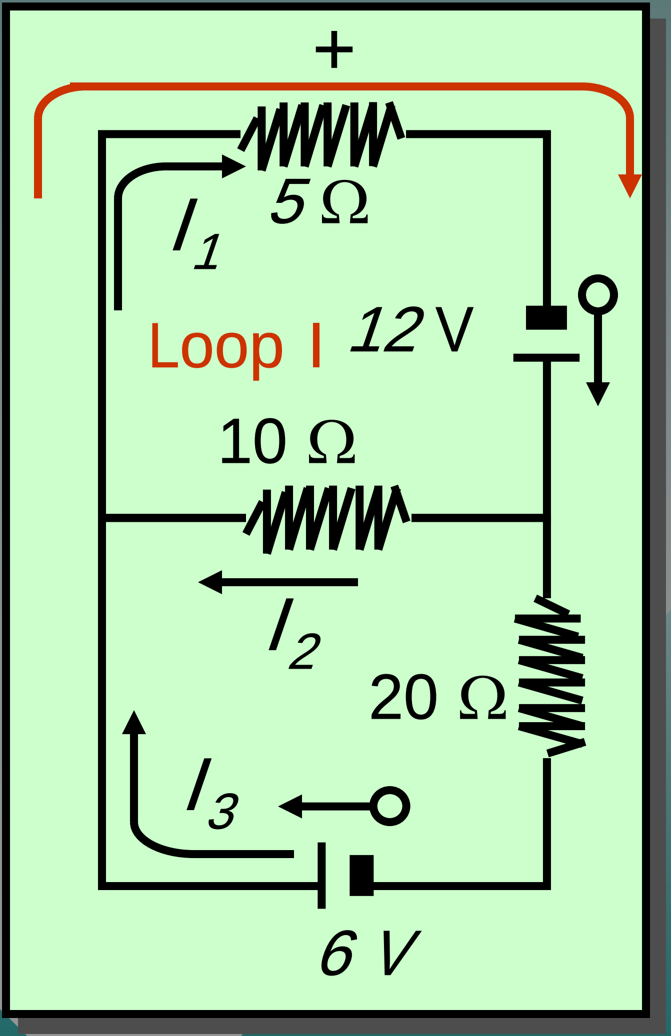
Consider Loop I tracing clockwise to obtain:

Voltage Rule: $\Sigma\mathcal{E} = \Sigma IR$

$$12\text{ V} = (5\ \Omega)\,I_1 + (10\ \Omega)\,I_2$$

Recalling that V/Ω = A, gives

$$5I_1 + 10I_2 = 12\text{ A}$$



Example 5 (Cont.) Finding the currents.

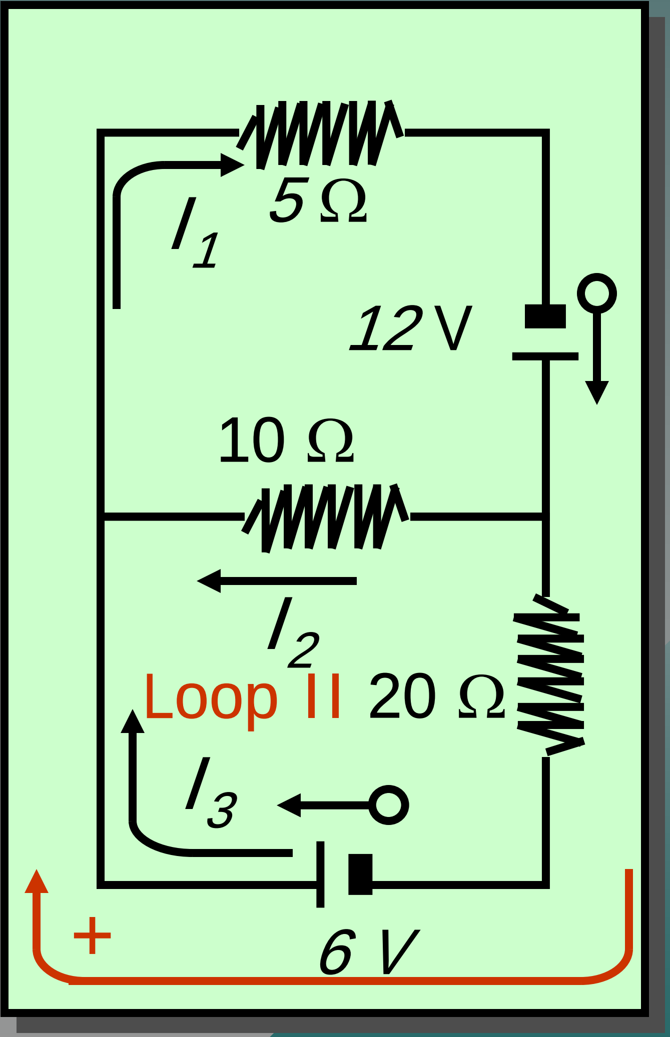
Consider Loop II tracing clockwise to obtain:

Voltage Rule: $\Sigma\mathcal{E} = \Sigma IR$

$$6\text{ V} = (20\ \Omega)I_3 - (10\ \Omega)I_2$$

Simplifying: Divide by 2 and V/Ω = A, gives

$$10I_3 - 5I_2 = 3\text{ A}$$

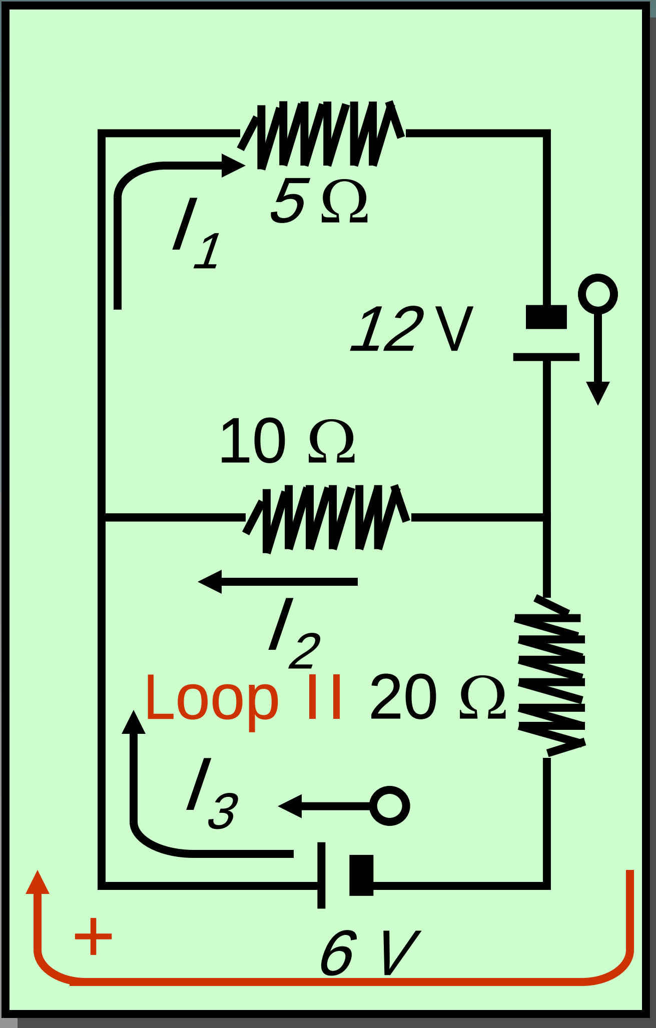


Example 5 (Cont.) Three independent equations can be solved for I_1, I_2, and I_3.

(1) $I_2 + I_3 = I_1$

(2) $5I_1 + 10I_2 = 12$ A

(3) $10I_3 - 5I_2 = 3$ A



Substitute Eq.(1) for I_1 in (2):

$$5(I_2 + I_3) + 10I_3 = 12 \text{ A}$$

Simplifying gives:

$5I_2 + 15I_3 = 12$ A

Example 5 (Cont.) Three independent equations can be solved.

(1) $I_2 + I_3 = I_1$

(3) $10I_3 - 5I_2 = 3$ A

(2) $5I_1 + 10I_2 = 12$ A

$15I_3 + 5I_2 = 12$ A

Eliminate I_2 by adding equations above right:

$$10I_3 - 5I_2 = 3 \text{ A}$$
$$15I_3 + 5I_2 = 12 \text{ A}$$

$$25I_3 = 15 \text{ A}$$

$I_3 = 0.600$ A

Putting $I_3 = 0.6$ A in (3) gives:

$$10(0.6 \text{ A}) - 5I_2 = 3 \text{ A}$$

$I_2 = 0.600$ A

Then from (1): $I_1 = 1.20$ A

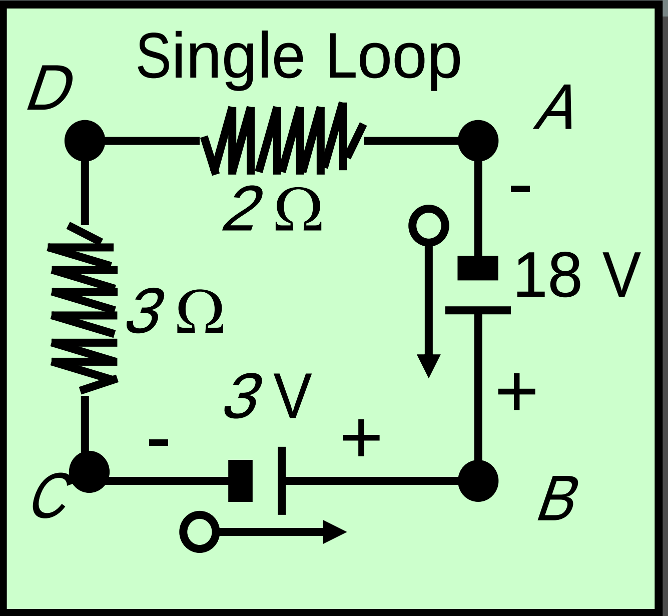
Summary of Formulas:

Rules for a simple, single loop circuit containing a source of emf and resistors.

Resistance Rule: $R_e = \Sigma R$

Current: $$I = \frac{\Sigma \mathcal{E}}{\Sigma R}$$

Voltage Rule: $\Sigma \mathcal{E} = \Sigma IR$



Summary (Cont.)

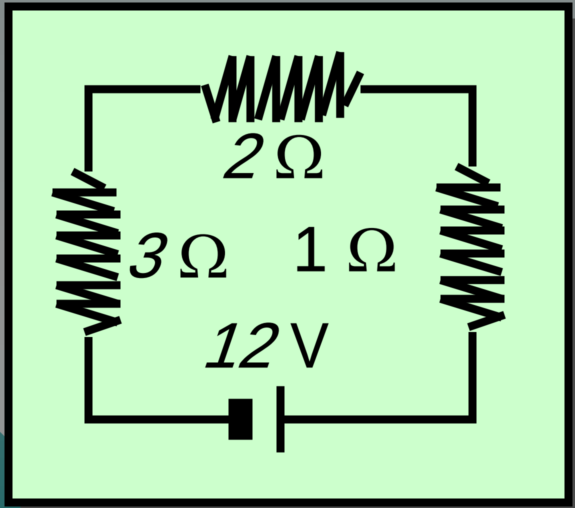
For resistors connected in series:

For series connections:

$$I = I_1 = I_2 = I_3$$
$$V_T = V_1 + V_2 + V_3$$

$$R_e = R_1 + R_2 + R_3$$

$$R_e = \Sigma R$$



Summary (Cont.)

Resistors connected in parallel:

For parallel connections:

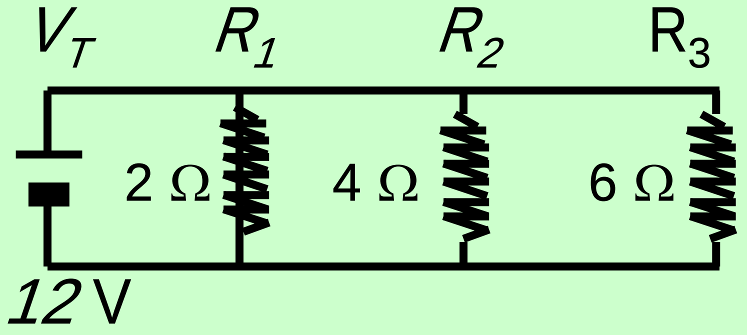
$$V = V_1 = V_2 = V_3$$

$$I_T = I_1 + I_2 + I_3$$

$$\frac{1}{R_e} = \sum_{i=1}^{N} \frac{1}{R_i}$$

$$R_e = \frac{R_1 R_2}{R_1 + R_2}$$

Parallel Connection



Summary Kirchoff's Laws

<u>Kirchoff's first law:</u> The sum of the currents entering a junction is equal to the sum of the currents leaving that junction.

Junction Rule: ΣI *(enter)* = ΣI *(leaving)*

<u>Kirchoff's second law:</u> The sum of the emf's around any closed loop must equal the sum of the IR drops around that same loop.

Voltage Rule: $\Sigma \mathcal{E} = \Sigma IR$

CONCLUSION: Chapter 28A Direct Current Circuits